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Downlink Beamforming with Delayed Channel Estimates under Total Power, Element Power and Equivalent Isotropic Radiated Power (EIRP) Constraints

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Abstract— A WLAN-type scenario where a base-station (access point) equipped with multiple antennas is transmitting and a mobile with only a single antenna is receiving, is considered. Three approaches are investigated: Grid-of-beams (GOB), Maximal-Ratio (MR) and equal-gain (EG), each with one of the following constraints 1) Total transmit power over all antenna elements, 2) Maximum power on any antenna element 3) Equivalent isotropic radiated power (EIRP). The median diversity gain for the MR approach was estimated to be 14.8dB, 10.6dB, and 11.4dB using constraint 1-3, respectively at 5.2GHz in a modern University building at 5-50 meter range. For GOB corresponding numbers are 9.4dB, 9.4dB, 2.3dB and for EG 13.9dB, 13.9dB, 10.3dB, respectively. These results were virtually independent of delay between channel estimation and use of the same estimate, when the delay is less than 130ms. This result was obtained although there were people moving in the environment.

The diversity gain under the EIRP constraint is encouraging and shows that coverage improvements are possible even under EIRP limitations.

I. INTRODUCTION

The use of multiple antennas in radio transmitters and receivers to improve link and/or system performance of a communications system has been analyzed in many papers. In the early papers, [9] and [1] uplink combining in base-stations equipped with antenna arrays in cellular contexts were addressed. Such algorithms and their analysis have later been refined in numerous papers e.g. [5] and [6]. Downlink transmission with antenna arrays at the base-stations has been treated using direction of arrival-type of information - in papers such as [3] and [10]. Transmission using knowledge of the transmission channel is treated in [4] and [2]. In the former paper, the channel knowledge is obtained from *explicit* feedback i.e. the mobile calculates the channel using special features of the air-interface (designed for this purpose) and feeds-back the information to the transmitter using encoded information, while in the latter the inherent feed-back obtained in a Time Division Duplex (TDD) system when employing calibrated receiver - transmitters is used. Although substantially different from an implementation point of view, one paramount parameter in the analysis of both schemes are the channel variations that occur during the time between channel estimation and the time of trans-

mission. This is one of the main topics of this paper. Transmission with antenna arrays without any channel information at the transmitter, has been frequently addressed recently, and a number of schemes collectively referred to as space-time coding have been developed [7, 8].

This paper addresses downlink beamforming in a low-mobility system, for example a wireless local area network (WLAN). Knowledge of the transmission channel such as in [2, 4] is assumed, although we leave the implementation open for explicit feedback or use of the TDD structure. Thus we consider an access point (base-station) equipped with multiple antennas. The mobile is assumed to have only a single antenna. Three approaches are considered: Grid-of-beams (GOB), Maximal-Ratio (MR) and equal-gain (EG). In the GOB approach, the base-station communicates through a set of fixed beams i.e. the strongest beam is selected for communication. In the MR approach the phase and amplitude between the multiple antennas at the base-station and the single antenna at the mobile is estimated (a flat-fading channel model is used) and the transmit weights (the scalar transmit signal is transmitted with different phase and amplitude weights in the antenna elements), are selected to maximize the power received at the mobile divided by the total employed transmit power. In the EG approach finally, the signal is transmitted with the same power on all antenna elements - only the phase is adjusted according to the channel.

None of the approaches attempt to do co-channel interference suppression. The reason is that the interference situation at the time of transmission is unknown in typical WLAN scenarios.

The performance criterion used to compare the approaches is the power received at the mobile as a function of the delay between channel-estimation and the transmission based on these estimates. Three different constraints on the transmit power are also investigated 1) Total transmit power, 2) Maximum power on any antenna element 3) Maximum equivalent isotropic radiated power (EIRP) (i.e. the power input to the antenna times the antenna gain in the direction of its maxima). The first constraint is reasonable if the interference pollution created by the transmission is important e.g. as in an interference limited multi-cell system. The second constraint is reasonable if there is a critical

¹ This report is a part of the work done within the IST-SATURN project. Zetterberg is also with A Brand New World in Sweden AB.

limitation of the power available from the amplifiers and range is of primary concern. The third condition finally, is a regulatory condition.

The performance of the three approaches under all three constraints will be obtained by means of calculations using measurement data. Note that the performance of all three approaches under all three criteria are of importance since we may want a solution that is all round i.e. has good coverage, interference and regulatory properties. Simulations using simple propagation models will also be performed. The author does not claim that the simulation model is very accurate - the simulations are merely included to illustrate some properties of the solutions.

The requirement on the delay between the channel-estimation/beam-selection time instant, and the time of transmission is of great importance from a system/standard point of view. In HIPERLAN-II for instance, a 2ms interval is the smallest possible. This small interval can be achieved by doing channel estimation from the preceding uplink (e.g. ARQ-feedback) burst. If a larger interval can be accepted (say 100-500ms) then it would be possible to implement a feedback scheme that would avoid phase-matching of the receiver/transmitter pairs. Such a scheme would typically employ a calibration burst where uncorrelated signals are transmitted on the antenna elements, and the mobile is in a position to estimate the channel and feedback the result. Another advantage is that the mobile is free to use different up- and down-link antennas which is not the case in the first - which could potentially be a limiting factor in some hardware designs of the mobile.

If seconds of delay can be tolerated, then the beam-selection can be made using the multi-sector support defined in e.g. the HIPERLAN-II standard using up to eight sectors.

In this paper we find based on measurements collected in a modern university building at 5.2dB a median diversity gain for the MR approach of 14.8dB, 10.6dB, and 11.4dB using constraints on total power, element power, and EIRP, respectively. For GOB corresponding numbers are 9.4dB, 9.4dB, 2.3dB and for EG 13.9dB, 13.9dB, 10.5dB. With a delay of 1.3 seconds the degradation is less than 1.4dB for all approaches under all constraints, even though there were people moving during the measurements! (the mobile antenna itself was stationary though).

The diversity gain under the EIRP constraint (11.4dB for MR) is an extremely encouraging results which shows that coverage improvements through adaptive antennas are possible even under EIRP limitations! Moreover, it may be the only approach which can substantially improve coverage under this constraint since it is no longer possible to improve range by "simply" using higher output power.

The paper is organized as follows. The mathematical framework is developed in Section II. The three approaches and the three constraints are then described in Section III and IV, respectively, using the mathematical framework of Section II. Simulation and measurement results are given in Section V. Finally, conclusions are drawn in Section VI.

II. PRELIMINARIES

Let the vector of antenna gain and phases, of the access point, in azimuth direction, θ , (we assume for simplicity that all signals arrive at zero elevation angle), be denoted $\mathbf{a}(\theta)$. Thus if a radio-

wave modulated by the signal, $s(t)$ arrives from angle θ_0 at the access point, the vector of signals, $\mathbf{x}(t)$, received at the output of the array is given by

$$\mathbf{x}^{\text{UL}}(t) = \mathbf{a}(\theta_0)s^{\text{UL}}(t), \quad (1)$$

if the signal source is located in the far-field. If the same signal is received from N different rays, each with an individual azimuth angle, θ_k , amplitude A_k and phase ϕ_k the received signal becomes

$$\mathbf{x}^{\text{UL}}(t) = \sum_{k=1}^N A_k \exp(j\phi_k) \mathbf{a}(\theta_k) s^{\text{UL}}(t) = \mathbf{v} s^{\text{UL}}(t), \quad (2)$$

where the channel vector \mathbf{v} is given by

$$\mathbf{v} = \sum_{k=1}^N A_k \exp(j\phi_k) \mathbf{a}(\theta_k). \quad (3)$$

This is effectively a flat fading assumption i.e. the channel is independent of frequency within the employed band-width. Since the angle, amplitude and phase of the rays change with time, the channel vector \mathbf{v} will be time dependent and we therefore modify (2) yielding

$$\mathbf{x}^{\text{UL}}(t) = \mathbf{v}(t)s^{\text{UL}}(t). \quad (4)$$

Since transmission is assumed to be performed at the same frequency as reception, reciprocity yields that the signal received at the mobile $s^{\text{DL}}(t)$ is given by

$$s^{\text{DL}}(t) = \mathbf{x}^{\text{DL},T}(t)\mathbf{v}(t), \quad (5)$$

where $(\cdot)^T$ denotes the transpose operation. Let the signal transmitted in the transmit antennas be given by $\mathbf{x}^{\text{DL}}(t) = \mathbf{w}x^{\text{DL}}(t)$ where \mathbf{w} is a vector of complex weights and $x^{\text{DL}}(t)$ is a scalar information carrying waveform, then (5) may be rewritten

$$s^{\text{DL}}(t) = \mathbf{w}^T \mathbf{v}(t)x^{\text{DL}}(t). \quad (6)$$

Assuming, without loss of generality, that $x^{\text{DL}}(t)$ has power one i.e. $E\{|x^{\text{DL}}(t)|^2\} = 1$, the power transmitted from the l th antenna is given by $P_l = |w_l|^2$ and the total transmit power consequently,

$$P_{\text{tot}} = \mathbf{w}^* \mathbf{w}. \quad (7)$$

The power received at the mobile is then given by

$$P_{\text{received-mobile}} = |\mathbf{w}^T \mathbf{v}(t)|^2. \quad (8)$$

III. THE THREE APPROACHES

A. Maximum-Ratio

In Maximum-Ratio transmission, the quotient between the power received at the mobile and the total transmitted power is maximized i.e.

$$\mathbf{w}_{\text{MR}} = \arg_{\mathbf{w}} \max \left\{ \frac{|\mathbf{w}^T \mathbf{v}(t)|^2}{\mathbf{w}^* \mathbf{w}} \right\}, \quad (9)$$

it is easily shown that the solution to this maximization is

$$\mathbf{w}_{\text{MR}} = \frac{\sqrt{P_{\text{tot}}}}{\|\mathbf{v}(t)\|} \mathbf{v}^c(t), \quad (10)$$

where $(\cdot)^c$ denotes complex conjugate, and P_{tot} is the total amount of transmit power. In reality $\mathbf{v}(t)$ has to be replaced by an estimate, furthermore there will be a delay from the time of estimation to the use of the estimate. Thus in practice we use

$$\hat{\mathbf{w}}_{\text{MR}} = \frac{\sqrt{P_{\text{tot}}}}{\|\hat{\mathbf{v}}(t - \Delta t)\|} \hat{\mathbf{v}}^c(t - \Delta t), \quad (11)$$

where Δt is the delay. The maximum transmit power in any antenna element is

$$P_{\text{element}} = \frac{P_{\text{tot}}}{|\hat{\mathbf{v}}(t - \Delta t)|^2} \max_l |v_l(t - \Delta t)|^2 \quad (12)$$

B. Grid-of-beams

The idea behind this approach is that propagation is concentrated around a certain angle of arrival i.e.

$$\mathbf{v}(t) \approx \text{constant} \times \mathbf{a}(\theta_0) \quad (13)$$

for some θ_0 . Using the same criterion as in maximal-ratio combining combined with (13) yields the following solution

$$\mathbf{w}_{\text{GOB}} = \frac{\sqrt{P_{\text{tot}}}}{\|\mathbf{a}(\theta_0)\|} \mathbf{a}^c(\theta_0), \quad (14)$$

The angle is now confined to a set of pre-defined angles Θ . This can be done without losing too much of performance compared with a continuous scan. This means effectively that we choose one beam in a set or grid of beams. Using (2) with the constraint (13) and replacing $\mathbf{v}(t)$ by $\hat{\mathbf{v}}(t - \Delta t)$ yields

$$\hat{\theta}_0 = \arg \max_{\theta \in \Theta} \frac{|\mathbf{a}^*(\theta) \hat{\mathbf{v}}(t - \Delta t)|^2}{\|\mathbf{a}^*(\theta)\|^2}. \quad (15)$$

C. Equal-Gain

In this approach we maximize the power received at the mobile divided by the maximum of the powers input to any antenna element i.e.

$$\mathbf{w}_{\text{EG}} = \arg \max_{\mathbf{w}} \left\{ \frac{|\mathbf{w}^T \mathbf{v}(t)|^2}{\max_l \{|w_l|^2\}} \right\}. \quad (16)$$

Defining a transformed weighting vector $\tilde{\mathbf{w}}$ through $\tilde{w}_l = \exp(j \arg\{v_l\}) w_l$ yields

$$\tilde{\mathbf{w}} = \arg \max_{\tilde{\mathbf{w}}} \left\{ \frac{|\sum_{l=1}^m \tilde{w}_l |v_l(t)|^2|^2}{\max_l \{|\tilde{w}_l|^2\}} \right\} \quad (17)$$

From (17) it is obvious that all elements of $\tilde{\mathbf{w}}$ should have the same argument e.g. zero. Thus $\tilde{\mathbf{w}}$ is taken to be a real-valued vector. With $\tilde{\mathbf{w}}$ real-valued it is also obvious that all antennas should transmit at the same power. Thus \mathbf{w}_{EG} is given by

$$\mathbf{w}_{\text{EG}} = \sqrt{P_{\text{element}}} \exp(-\arg\{\mathbf{v}(t)\}), \quad (18)$$

where P is the element transmit power. The total utilized power is given by

$$P_{\text{tot}} = m P_{\text{element}} \quad (19)$$

Again the actually employed transmit vector has to be based on a delayed estimate thus,

$$\hat{\mathbf{w}}_{\text{EG}} = \sqrt{P_{\text{element}}} \exp(-\arg\{\hat{\mathbf{v}}(t - \Delta t)\}). \quad (20)$$

IV. CONSTRAINTS

As was discussed already in the introduction the three approaches will be considered under constraints on, total power, element power and EIRP. The total power and element power constraints are obtained by simply letting

$$P_{\text{tot}} = P_{\text{tot-max-allowed}} \quad (21)$$

and

$$P_{\text{element}}(m) = \frac{P_{\text{element-max-allowed}}(1)}{m}, \quad (22)$$

respectively, where we have introduced a dependence on the number of the number of antennas, m , in order to compare systems with different number of antennas in a fair way.

The equivalent isotropic power is defined as the power input to the antenna times the antenna gain in the direction of its maxima. Transmitting with weights \mathbf{w} can be seen as exciting an antenna with power P_{tot} and antenna pattern $a_{\text{combined}}(\theta) = \frac{1}{P_{\text{tot}}} |\mathbf{w}^* \mathbf{a}(\theta)|^2$. Thus the EIRP is given by

$$\text{EIRP} = \max_{\theta} |\mathbf{w}^* \mathbf{a}(\theta)|^2, \quad (23)$$

and the EIRP constraint is obtained by letting $\text{EIRP} = \text{EIRP}_{\text{max-allowed}}$.

V. RESULTS

In this section we compare the performance of the three approaches, GOB, MR and EG, under constraints on total power, element power and EIRP. This is done for both simulated channels - to illustrate the dependence on spatial and temporal variability, and measured channels - to obtain actual performance. The measurement data have been collected by and at Bristol University, as a part of the SATURN project (www.ist-saturn.org). The antenna array in both simulations and the real experiments is a eight element linear array with directional antenna elements, with antenna spacing of half a wavelength. The element *amplitude* diagrams are assumed to be given by $p(\theta) = \cos^{1.8}(\theta)$ both in simulations and measurements. These diagrams fits the measured element patterns quite well. In the measurement data processing, the antenna diagram only comes into play when calculating performance under the EIRP constraint. The measurements have been made within in a modern university building at 5.2GHz. The distance between the base-station array and the mobile is between five and fifty meter.

A. Simulations

In this section we use a flat-Rayleigh fading channel model with Gaussian angle spread, σ , and classical Jakes Doppler spectrum with frequency f_d . The relevance of this model for the considered application is questionable - the results are merely

included for illustrative purposes. Figure 1 below show the cumulative distribution (CDF) of the power received at the mobile i.e. the distribution of $P_{\text{received-mobile}}$, see equation (8). The weighting vector \mathbf{w} are obtained from (11), (14-15) and (20) for the MR, GOB and EG approach, respectively, all using the total power constraint (21) and a 92ms estimation delay. Other assumptions are that the angle spread, σ is assumed to be fifteen degrees, $\sigma = 15^\circ$, the Doppler frequency is $f_d = 1\text{Hz}$. There are eight antenna elements with a spacing of a half a wavelength in the transmit array. Also shown in Figure 1 is the performance obtained using only a single antenna element. The ten percent level of the CDF is a reasonable point to compare the signal distributions for systems that employ re-transmission of lost bursts. The difference between the 10%-levels of one and multi-element transmission is termed the 10%-diversity gain. This number will be used as performance measure subsequently. In the simulation example of Figure 1 the 10%-diversity gain of the MR-, GOB- and EG-approaches are 14.1, 9.6 and 13.1 dB, respectively.

In Figure 2 we have plotted the 10%-diversity gain as a function of angle spread, σ , and the Doppler frequency time delay product, $f_d\Delta t$ for the MR approach. Total power, maximum element power and EIRP constraints are used in the upper- middle- and lower part of the viewgraphs, respectively. The results show that the highest performance is obtained at a large angle spread, σ , and a small frequency times delay product, $f_d\Delta t$. When this product increases the performance decays rapidly for large angle spreads and slowly for small. This shows that for an environment with lots of multipath the update rate must be very high, which is not the case in an environment with few multipaths. The results also shows, remarkably, that substantial performance gains under an EIRP constraint can also be achieved.

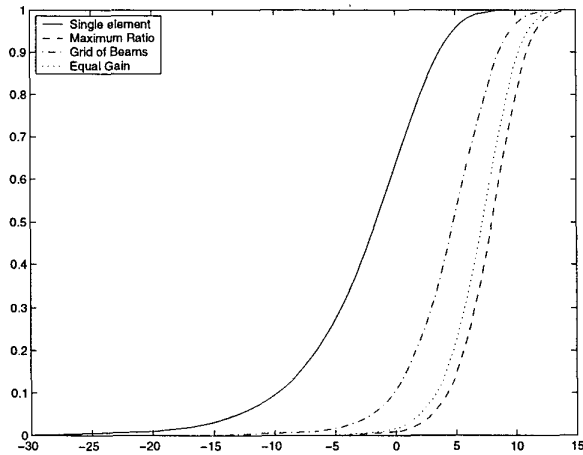


Fig. 1. Simulated performance of MR, GOB and Equal Gain under a total power constraint.

B. Measurements

The corresponding results, as the simulations, are also obtained from the SATURN measurement campaign at Bristol University. In this campaign the channel estimates are available for $15 \times 3 = 45$ RX-TX locations. The receiver has eight antennas and represents the access point, while the transmitter has just a single antenna and represents the mobile. We use the im-

pulse responses measured in uplink to predict the “would-be” downlink performance. In each location, channel estimates are available at 1599 time instances separated by 13.3ms in time. During the measurements at one location the RX and TX antenna positions remain fixed while the environment is changing due to occasionally moving people. For each of the 1599 time instances, channel estimates of 97 narrow-band frequency sub-channels are available. The results are obtained such that one sub-channel is selected at a time and the three beamformers, MR, GOB and EG are applied using an estimation delay which is an integer number times 13.3ms. The CDF of the signal distribution is then obtained by combining the results from all the frequency sub-channels. Figure 3 below show the CDFs using 13.3ms delay and the total power constraint, from an RX-TX location (RX 1, TX 2) where the distance between the mobile and the access point was fifty meters and the mobile is inside another room than then the access point. From Figure 3 we find that the 10%-diversity gain is 16.6dB, 11.4dB, and 15.3dB, using the MR-, GOB- and EG-approach, respectively.

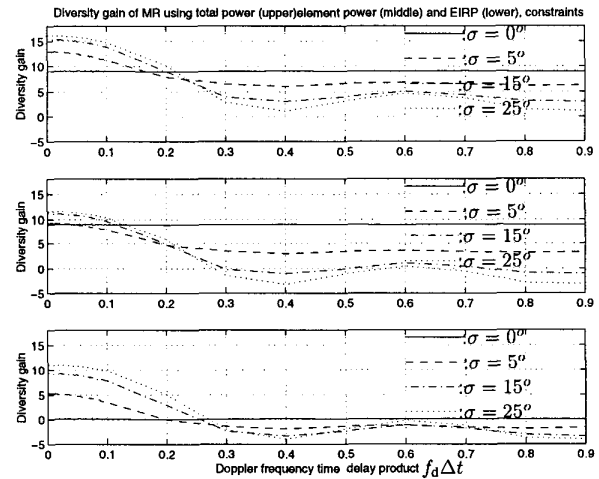


Fig. 2. Performance of MR as a function of frequency time delay product $f_d\Delta t$ using a propagation model. The upper, middle and lower sub-plot corresponds to total power, element power and EIRP constraints, respectively.

Figure 4 shows the max, median and min 10%-diversity gain as a function of delay for the MR approach. The upper, middle and lower plots correspond to total power-, element power- and EIRP-constraints, respectively. The realizations are quite evenly spread between the median and the extreme values. We note that the median diversity gains are of 14.8dB, 10.6dB, and 11.4dB for the MR approach using constraints on total power, element power, and EIRP, respectively with zero delay. For GOB the corresponding numbers are 9.4dB, 9.4dB, 2.3dB and for EG 13.9dB, 13.0dB, 10.3dB, respectively. With delays of 13.3ms, 133ms, and 1331ms the median performance degrades between 0.2-0.5dB, 0.4-0.7dB, and 1.0-1.4dB, respectively.

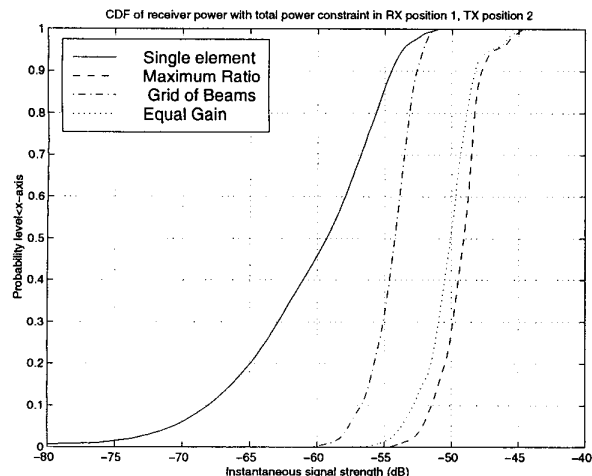


Fig. 3. Distribution of signal strength using MR, GOB and EG using the total power constraint, obtained from measurement data.

VI. CONCLUSION

The measurement results of Section V-B show that the diversity gains degraded with less than 1.4dB when the delay increased from 0 to 1331ms. This must imply that the channel non-stationarity caused by the movement of people in the measurements can be neglected. However, the antenna used during the measurements was mounted on a pole, and this may not be representative of e.g. a notebook application since in such a case the movements of the antenna caused by the user may come into play - this needs to be clarified by future studies. The results are encouraging and may for some applications (e.g. a stationary manufacturing machine with a WLAN modem) be sufficient. For non-fixed users the constraints of the simulations in Section V-A come into play. These results show that only a small performance reduction is experienced at a Doppler frequency times delay product of 5%. Assuming a 5.2GHz carrier frequency and 2ms delay - which are representative for a HIPERLAN-II application where the TDD structure is utilized as feedback - this corresponds to a speed of 5km/h. Increasing the delay to 100ms further reduces the maximum speed down to 0.1km/h. As was discussed in the introduction of this paper a delay on the order of 2ms makes an implementation which utilizes the TDD structure of the air interface for feed-back a more realistic alternative than feed-back.

The highest diversity gain under the total power constraint is obtained using MR which achieves 14.8dB on the measured channels (median value). However, for an element power constraint MR is 2.4dB inferior to EG which is also only 0.9dB worse than MR under the total power constraint. The GOB approach is notably worse but may be justified in some cases where the implementation cost is much lower.

The most remarkable result in this paper anyhow is the performance under EIRP constraint. The measurement results show a 10-12dB performance gain for MR and EG. This means that it is possible to extend range in EIRP limited bands such as the ISM- and the HIPERLAN-II allocation, using adaptive antenna techniques. (The GOB technique does not work in this constraint). This is in sharp contrast to what is generally thought

in the wireless-community. Moreover, it may be the only technique which can be used to substantially increase range of EIRP limited systems without sacrificing channel capacity! This is because the door is closed for "simply" increasing the transmit power in EIRP limited applications.

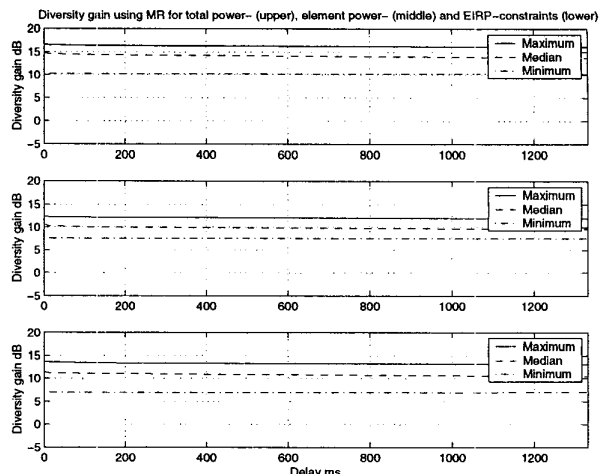


Fig. 4. Performance of MR as a function of delay using measurement data. The upper, middle and lower sub-plot correspond to total power, element power and EIRP constraints, respectively.

REFERENCES

- [1] S Anderson, M Millnert, M Viberg, and B Wahlberg. "An Adaptive Array for Mobile Communication Systems". *IEEE Transactions on Vehicular Technology*, vol. 40, pp. 230-236, February 1991.
- [2] J.M Cavers. "Multiuser Transmitter Diversity Through Downlink Beamforming". In *IEEE Wireless Communications and Networking Conference*, pp. 21-24, New Orleans, USA, September 1999.
- [3] D Gerlach. "Adaptive Transmitting Antenna Arrays at the Base Station in Mobile Radio Networks". PhD thesis, Information Systems Laboratory, Stanford University, Stanford CA 94305, USA, 1995.
- [4] D Gerlach and A Paulraj. "Adaptive Transmitting Antenna Arrays with Feedback". *IEEE Signal Processing Letters*, vol. 1, no. 10, pp. 150-152, October 1994.
- [5] J Karlsson and J Heinegård. "Interference Rejection Combining for GSM". In *International Conference on Universal Personal Communications*, pp. 433-437, Cambridge, USA, September 1996.
- [6] G Klang and Björn Ottersten. "Structured Semi-Blind Interference Rejection in Dispersive Multichannel Systems". *IEEE Transactions on Signal Processing*, 2000.
- [7] V. Tarokh, H. Jafarkhani, and A.R Calderbank. "Space-Time Block Codes from Orthogonal Designs". *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456-1467, July 1999.
- [8] V. Tarokh, N. Seshadri, and A.R Calderbank. "Space-Time Codes for High Data Rate Wireless Communication". *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744-765, March 1998.
- [9] J.H Winters. "Optimum Combining in Digital Mobile Radio with Cochannel Interference". *IEEE Transactions on Vehicular Technology*, vol. 33, no. 3, pp. 144-155, August 1984.
- [10] Per Zetterberg. "A Comparison of Two Systems for Downlink Communication with Base Station Antenna Arrays". *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1356-1370, September 1999.